OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Friday **21 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Where a numerical value for the acceleration due to gravity is needed, use g = 9.8 m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

1 Each of two light elastic strings, AB and BC, has modulus 20 N. AB has natural length 0.5 m and BC has natural length 0.8 m. The strings are both attached at B to a particle of mass 0.75 kg. The ends A and C are fixed to points on a smooth horizontal table such that AC = 2 m, as shown in Fig. 1.

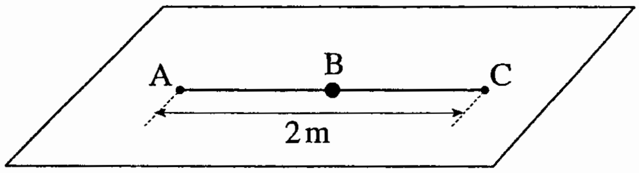


Fig. 1

Initially the particle is held at the mid-point of AC and released from rest.

(i) Find the tension in each string before release and calculate the acceleration of the particle immediately after it is released. [5]

The particle is now moved to the position where it is in equilibrium. The extension in AB is e m.

(ii) Calculate e. [4]

The particle is now held at A and released from rest.

(iii) Show that in the subsequent motion BC becomes slack. Calculate the furthest distance of the particle from A. [6]

2 A simple pendulum consists of a light inextensible string AB of length l with the end A fixed and a particle of mass m attached to B. The pendulum oscillates with period T.

(i) It is suggested that T is proportional to a product of powers of m, l and g. Use dimensional analysis to find this relationship. [4]

The angle that the string makes with the downward vertical at time t is θ. The particle is released from rest with the string taut and $\theta = \theta_0$.

(ii) Use the equation of motion of the particle to find the angular acceleration, $\ddot{\theta}$, in terms of θ, l and g. [3]

The angle θ_0 is chosen so that θ remains small throughout the motion.

(iii) Use the small angle approximation for $\sin\theta$ to show that the particle performs approximate angular simple harmonic motion. State the period of the motion and verify that it is consistent with your answer to part **(i)**. [4]

(iv) Calculate the proportion of time for which the particle travels faster than half of its maximum speed. [4]

3 Michael is attempting to make a small car do a 'loop-the-loop' on a smooth toy racing track. He propels a car of mass m kg towards a section of the track in the form of a vertical circle of radius 0.2 m and the car enters the circle at its lowest point with a speed of $2.8\,\mathrm{m\,s^{-1}}$. During the motion around the circle the angle the car has turned through is denoted by θ, as shown in Fig. 3.

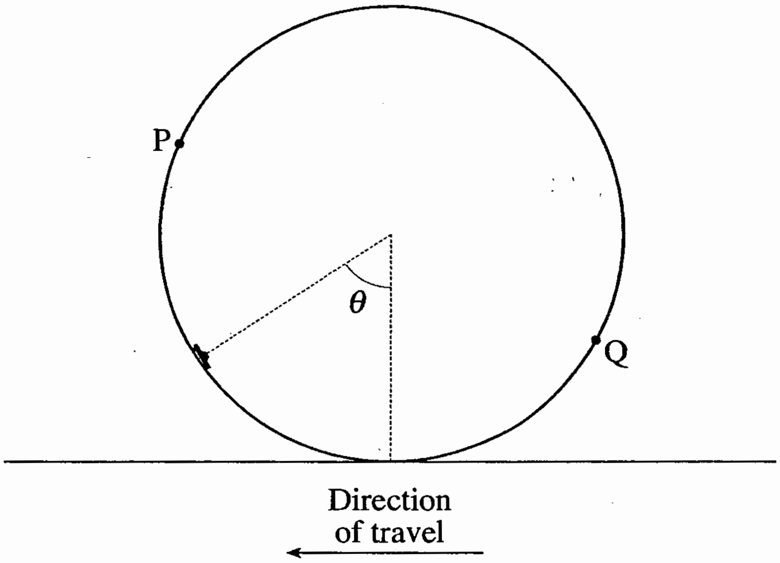


Fig. 3

(i) Show that the speed, $v\,\mathrm{m\,s^{-1}}$, of the car is given by $v^2 = 3.92(1 + \cos\theta)$. Hence show that the reaction of the track on the car, R N, is given by $R = 9.8m(2 + 3\cos\theta)$. [7]

The car leaves the track at the point P where $\theta = \alpha$.

(ii) Calculate α. [2]

(iii) Calculate the speed of the car at P and hence calculate the greatest height of the car above the level of P. [3]

The car hits the track at the point Q which is $\frac{22}{135}$ m below the level of the centre of the circle.

(iv) Calculate the speed with which the car hits the track at Q. [3]

4 Fig. 4.1 shows a uniform lamina OAB in the shape of the region between the curve $y = 4x - x^3$ and the x-axis. The point G is the centre of mass of the lamina.

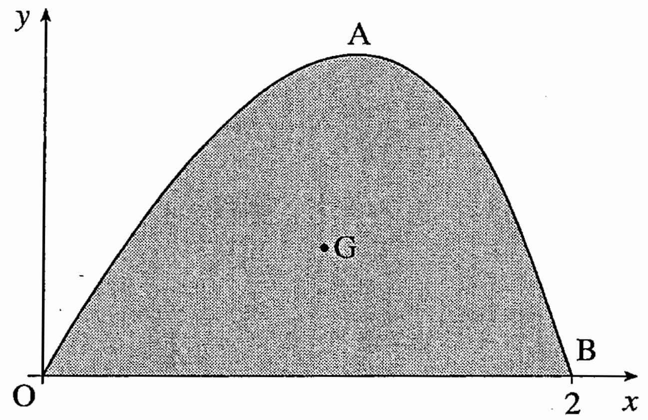


Fig. 4.1

(i) Show that G has coordinates $(\frac{16}{15}, \frac{128}{105})$. [11]

OAB is suspended by wires at O and B and hangs in equilibrium in a vertical plane with OB horizontal. The wire at B is at 60° to the horizontal and the wire at O is at α° to the horizontal, as shown in Fig. 4.2.

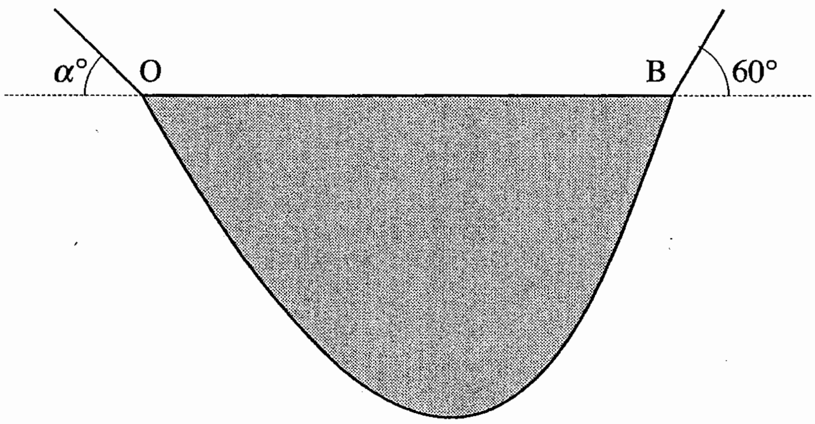


Fig. 4.2

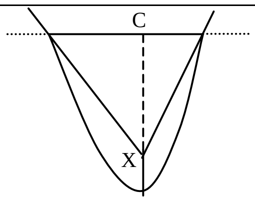
(ii) Calculate α. [4]

Mark Scheme

1(i)	$T_{AB}=\frac{20\times 0.5}{0.5}=20$ N	M1 A1	Hooke's law	
	$T_{BC}=\frac{20\times 0.2}{0.8}=5$ N	A1		
	$(\pm)0.75a=20-5$	M1	N2L	
	$a=(\pm)20\text{ m s}^{-2}$	F1		
				5
(ii)	$\frac{20e}{0.5}=\frac{20(0.7-e)}{0.8}$	B1 M1 A1	$0.7-e$ equilibrium equation	
	$e=0.269$	A1	cao	
				4
(iii)	if BC goes slack when speed = v			
	$\frac{1}{2}mv^2+\frac{\lambda\times 0.7^2}{2\times 0.5}=\frac{\lambda\times 1.2^2}{2\times 0.8}$	M1	energy equation	
	$\frac{1}{2}mv^2=8.2>0$ hence BC goes slack	E1		
	at furthest distance, $\frac{\lambda\times(\text{AB}-0.5)^2}{2\times 0.5}=\frac{\lambda\times 1.2^2}{2\times 0.8}$	M1 M1 A1	energy equation EPE term in terms of a variable correct equation	
	AB = 1.45 m	A1	cao	
				6

2(i)	$T=km^{\alpha}l^{\beta}g^{\gamma}\Rightarrow \text{T}=\text{M}^{\alpha}\text{L}^{\beta}(\text{LT}^{-2})^{\gamma}$	M1	substitute dimensions	
	$\alpha=0$	M1	equate indices and solve	
	$-2\gamma=1\Rightarrow\gamma=-\frac{1}{2}$	A1	at least two correct indices	
	$\beta+\gamma=0\Rightarrow\beta=\frac{1}{2}$ so $T=k\sqrt{\frac{l}{g}}$	A1	formula (aef)	
				4
(ii)	$ml\ddot{\theta}=-mg\sin\theta$	M1 A1	N2L tangentially (no tension term) accept sign error	
	$\ddot{\theta}=-\frac{g}{l}\sin\theta$	A1		
				3
(iii)	$\ddot{\theta}\approx-\frac{g}{l}\theta$ hence SHM	M1	use $\sin\theta\approx\theta$	
		E1	must conclude SHM	
	period $=2\pi\sqrt{\frac{l}{g}}$	B1	follow their SHM equation	
	i.e. as in (i) with $k=2\pi$	B1		
				4
(iv)	$\dot{\theta}=-\theta_0\omega\sin\omega t$	M1	clear attempt at velocity (not displacement) in terms of time (must use sin or cos)	
	$\lvert\dot{\theta}\rvert>\frac{1}{2}\dot{\theta}_{\max}\Leftrightarrow\lvert\sin\omega t\rvert>\frac{1}{2}$	M1	inequality for sin (or cos)	
	for one half-period, $\left(0\leqslant t\leqslant\frac{\pi}{\omega}\right)$ we have $\frac{1}{6}\pi<\omega t<\frac{5}{6}\pi$	M1	solve inequality	
	proportion $=\frac{\frac{5\pi}{6\omega}-\frac{\pi}{6\omega}}{\frac{\pi}{\omega}}=\frac{2}{3}$	A1		
				4

3(i)	$\frac{1}{2}m\cdot 2.8^2 - mg\cdot 0.2 = \frac{1}{2}mv^2 - mg\cdot 0.2\cos\theta$	M1	attempt energy equation	
	$v^2 = 2.8^2 - 0.4g + 0.4g\cos\theta$	A1	correct equation (any form)	
	$v^2 = 3.92(1+\cos\theta)$	E1		
	$R - mg\cos\theta = m\dfrac{v^2}{r}$	M1 A1	N2L with $\frac{v^2}{r}$ or $r\omega^2$	
	$= 19.6m(1+\cos\theta)$	M1	substitute v^2	
	$R = 9.8m(2+3\cos\theta)$	E1	must use N2L in form $\sum F = ma$ or clearly justify signs	7
(ii)	leaves when $R = 0$	M1		
	$2+3\cos\theta = 0 \Rightarrow \alpha = \cos^{-1}\left(-\frac{2}{3}\right) \approx 2.3\text{rad} \approx 132°$	A1		2
(iii)	$v^2 = 3.92(1-\frac{2}{3}) \Rightarrow v = \sqrt{\dfrac{3.92}{3}} \approx 1.143$	B1		
	vertical cpt $v_y = \sqrt{\dfrac{3.92}{3}}\sin(\pi-\alpha)$	M1		
	${v_y}^2 = 2gh \Rightarrow h = \frac{1}{27} \approx 0.037$ m	A1		3
(iv)	$\frac{1}{2}mv^2 - mg\cdot\frac{22}{135} = \frac{1}{2}m\cdot 2.8^2 - mg\cdot 0.2$	M1 A1	energy equation	
	$v = 2.67$ m s^{-1}	A1	cao	3

4(i)	area $= \int_0^2 (4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2$	M1	calculate area	
	$= 4$	A1		
	$4\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}\int_0^2 xy\,dx\\ \int_0^2 \frac{1}{2}y^2dx\end{pmatrix}$	B1 B1	$\int xy$ or $\int \frac{1}{2}y^2dx$ seen both formulae correct	
	$4\bar{x} = \int_0^2 \left(4x^2 - x^4\right)dx$	M1	integrate their expression	
	$= \left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2$	A1	or multiple	
	$4\bar{y} = \int_0^2 \frac{1}{2}\left(16x^2 - 8x^4 + x^6\right)dx$	M1	integrate their expression	
	$= \frac{1}{2}\left[\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7\right]_0^2$	A1	or multiple	
		M1	limits	
	$\bar{x} = \frac{16}{15}$	E1		
	$\bar{y} = \frac{128}{105}$	E1		11
(ii)	 Forces concurrent	B1	(or correct moments equation)	
	$CX = \frac{14}{15}\tan 60°$	M1	(or attempt two other equations)	
	$\frac{16}{15}\tan\alpha = CX = \frac{14}{15}\sqrt{3}$	M1	(or eliminate tensions)	
	$\Rightarrow \alpha = 56.6°$	A1		4

Examiner’s Report